Mathematics Calculation Policy

Rationale

This policy has been designed to show progression in written mathematical methods throughout the school. Our written calculation policy is set out to show:

- The objectives stipulated for all four operations by the National Curriculum
- The calculation methods for each year group
- Relevant vocabulary needed at each stage
- Examples of reasoning activities

Concrete, Pictorial and Abstract

Each method has examples as to what it looks like in the concrete, pictorial and abstract forms. All learners are introduced to a calculation method for the first time using concrete manipulatives. Concrete resources from EYFS to Year 6 include: bead strings, Base 10, Cuisenaire Rods, Place Value counters, Numicon and Snap cubes. Children will then progress through to a pictorial stage before moving to the abstract. During the pictorial stage, children will be taught to use the bar model (see separate policy). The amount of time needed to progress through each stage is unique to each learner.

Mastering Calculation

The new curriculum has a strong focus on mastery and therefore, if a child is fluent in a method for their year group, they should not be moved onto a different method of calculation or a larger set of numbers (see the quick glance guides in this policy). Instead, children will be encouraged by their teacher to 'go deeper' within this method. This may involve: using it in different contexts; using and applying it to other learning; using it with missing digits or values; explaining or experimenting with different aspects of it; proving answers with pictures or manipulatives; or explaining what has gone wrong in a calculation. Children must also check their calculations through the use of estimation and inverse operations.

Mathematical Vocabulary

The National Curriculum places great emphasis upon the use of correct mathematical vocabulary and children developing this. Throughout school, children are strongly recommended to use and apply mathematical vocabulary when learning a new method or concept. They will be constantly exposed to this, have it expertly modelled by their teacher and be expected to use it themselves when justifying methods.

Mental Methods

Children should always be encouraged to see if they can work out a calculation mentally before trying a written method. Children will be shown number patterns and relationships between numbers throughout the school. Times tables are introduced and taught in specific year groups: Reception: x2; Year 1: x5, x10; Year 2: x4; Year 3: x3, x6, x8; Year 4: x7, x9, x11, x12.

Mathematical Vocabulary

Year 1	+	-	x	÷
Mathematical vocabulary	count on, count back, number bonds, number facts, subtraction facts, fact family, add, subtract, more, less, plus, minus, total, sum, difference between, equal		grouping, sharing, multiply, divide, double, half, array, lots of	

Year 2	+	-	x	÷
Mathematical vocabulary	Add, subtract, count on, count back, more, less, plus, minus, total, sum, difference, partition, bridge, round, inverse, number line, number facts, multiple of 10, regroup		Inverse, operation, multiplication table, times table, multiply, multiplication, times, product, repeated addition, lots of, array, divide, division, shared by, halve, double	

Year 3	+	-	x	÷
Mathematical vocabulary	Add, subtract, count on, count back, more, less, plus, minus, total, sum, difference, partition, bridge, round, inverse, number facts, multiple of 10, regroup		Inverse, operation, multiplication table, times table, multiply, multiplication, times, product, repeated addition, lots of, array, divide, division, shared by, halve, double	

Year 4	+	-	x	÷
Mathematical vocabulary	addition, subtraction, sum, total, difference, minus, less, plus, altogether, column addition, column subtraction, regroup, operation, estimate, equal, method, inverse		place value, multiply, multiplication, times, product, divide, division, factor, factor pairs, multiplication & division facts, operation, estimate, multiple, shared equally, array	

Year 5	+	-	x	÷
Mathematical vocabulary	addition, subtraction, sum, total, difference, minus, less, column addition, column subtraction, operation, regroup, inverse, estimate, digit, place holder, rounding, approximate, accuracy		multiply, multiplication, times, product, commutative, short multiplication, long multiplication, multiplication facts, estimate, multiple, remainder	

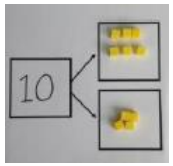
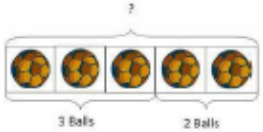
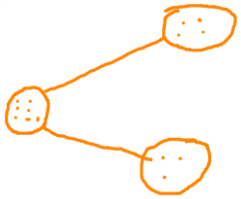
Year 6	+	-	x	÷
Mathematical vocabulary	addition, subtraction, sum, total, difference, minus, less, column, operation, inverse, estimate, approximate, multiply, multiplication, times, product, commutative, short multiplication, long multiplication, estimate, remainder, fraction, decimal, divisible			

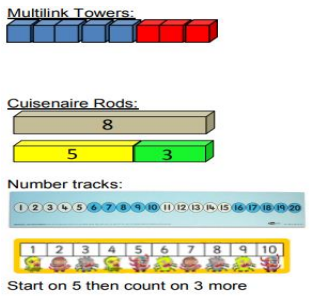
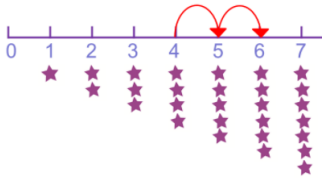
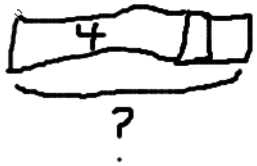
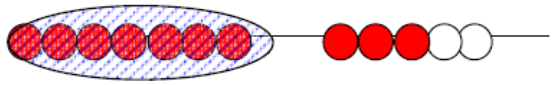
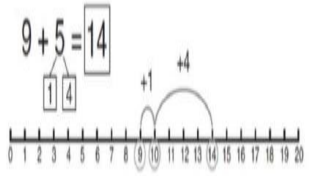
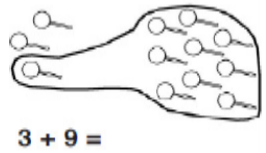
Progression in Written Calculation

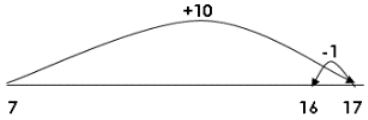
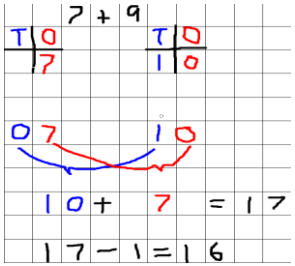
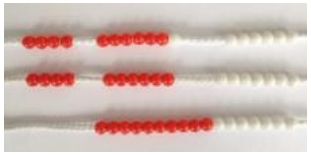
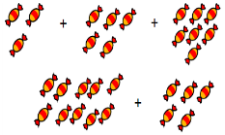
Addition

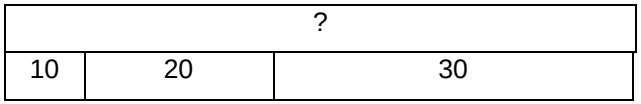
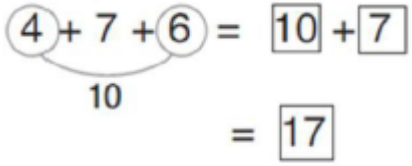
Addition and Subtraction are connected. Addition names the whole in terms of parts, while subtraction names a missing part of the whole.

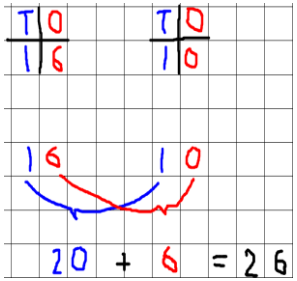
Part	Part
Whole	

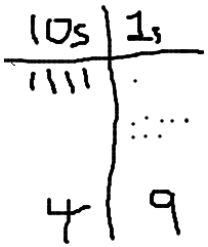
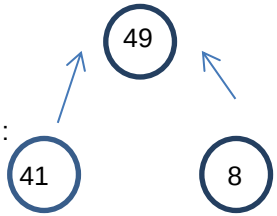
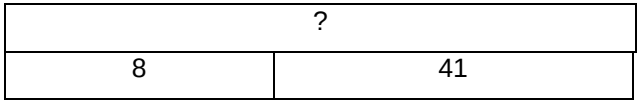
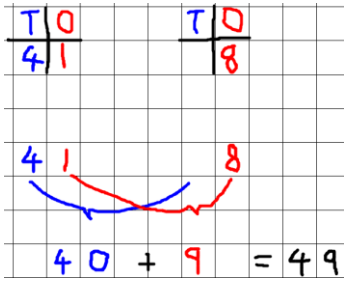
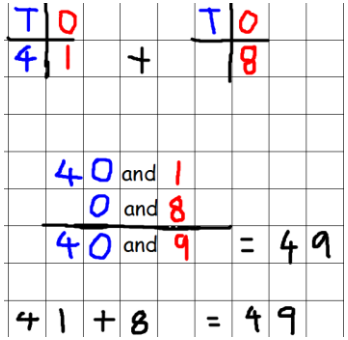
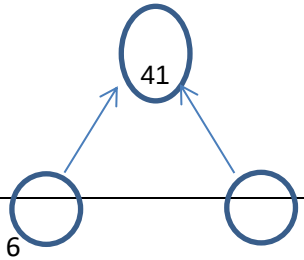
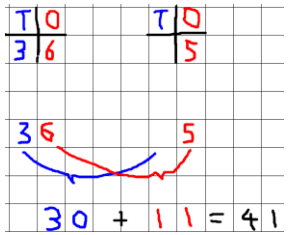
Objective and strategies	Concrete	Pictorial	Abstract
EYFS/Year 1			
Combining two parts to make a whole: part-whole model	   Use cubes to add two numbers together as a group or a bar (aggregation). Start counting at 1. Count one set and then the other. Then count them altogether. (Use other resources – eggs, shells, cars etc)	 Use pictures to add two numbers together.  Use the bar model to add two numbers together.  Children draw crosses, dots or numbers in a part-whole model and add together. 	Children start to show recognisable abstract number sentences alongside the pictorial and concrete creations. 4 + 3 = 7 10= 6 + 4 (Equal sign does not have to come at the end).

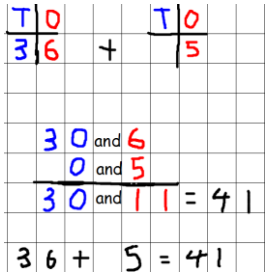
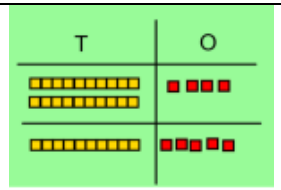
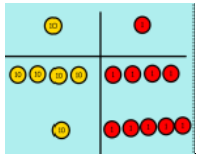
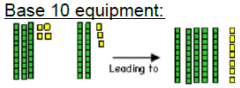
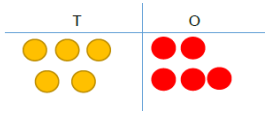
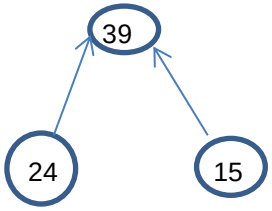
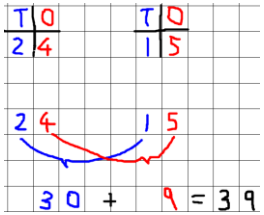
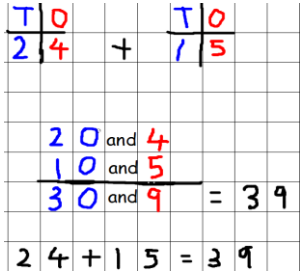
Starting at the bigger number and counting on	**This stage is essential.** Children start to calculate rather than just count. Where one quantity is increased by some amount (augmentation).  Count on from the total of the first set (3 in your head) and count on 2. Always start with the larger number. Use bead strings or Cuisenaire Rods. Number tracks teach children the order of numbers. Number line - points are marked instead (allowing fractions of numbers). Could write number sentence along with creation.	Use a number line with pictorial representation– start at the larger number and count on in ones.  Use a bar model that encourages the children to count on rather than count the whole.  This is an important moment as number lines are very different from number tracks.	Children start to show recognisable abstract number sentences. The sum is 4 + 2 = Or, = 4 + 2 Not, 2 + 4
Regrouping to make 10	Use ten frames and counters/cubes or use Numicon. 6+5 = 11 Use bead strings to show 7 + 5 can be partitioned into 7 + 3 + 2 (children use number bonds to 10). 	Children then draw the ten frame Use a number to partition (decompose) e.g. 9 + 5  Or, use their own pictures to show regrouping 	The sum is 9 + 5 = Children develop an understanding of equality: 6 + ? = 11 6 + 5 = 5 + ? 6 + 5 = ? + 4

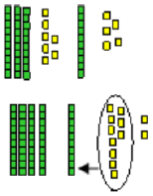
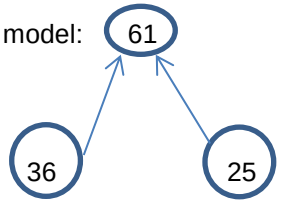
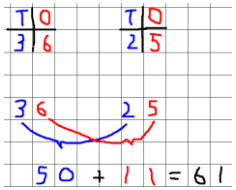
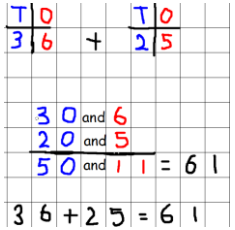
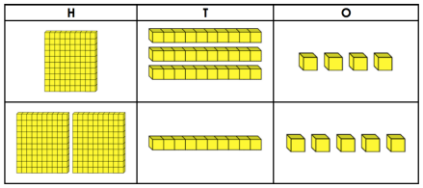
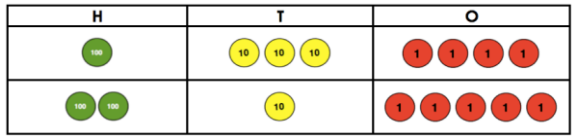
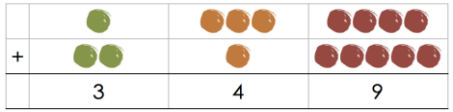
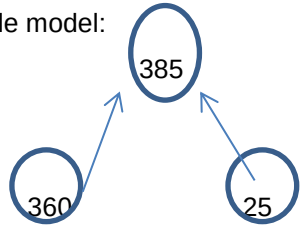
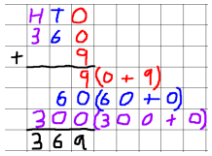
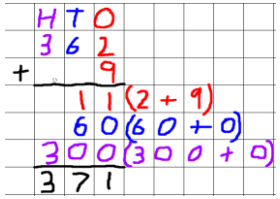
<table>
<tr><td>Adding 10 and then compensating</td><td>Use a bead string to work out sums
e.g. 7 + 9
 
Children find 7, then add 10 and then adjust by removing 1.</td><td>Children draw a picture to show the compensation.
 
</td><td>Introduce the informal partitioning method:
  </td></tr>
<tr><td>Adding in any order</td><td>Explore the commutative law, where you can add in any order, using cubes.</td><td>Draw the cubes to show that the total has not changed.</td><td>6 + 3 + 4 is the same as 6 + 4 + 3 however the second number sentence is easier & quicker.</td></tr>
<tr><td colspan="4">Year 2</td></tr>
<tr><td>Adding three single digits</td><td>Use bead strings to work out sums e.g.
4 + 7 + 6 = 17. Put 4 and 6 together to make 10. Add on 7.
 
Build a tower of bricks and then ask the child to split them in 3 ways – add the numbers together.</td><td>Add together three groups of objects. Draw a picture to recombine the groups to make 10.
  or 

Bar models represent the 3 numbers.
 
</td><td>Combine the two numbers to make 10 and then add on the remainder.
 

Children should be shown missing number sentences.</td></tr>
<tr><td>Adding 10</td><td>Use cubes or dienes to start with a number and add on 10 more.</td><td>Use or draw number squares to count on 10 more by looking at the number directly below.</td><td>Children to use informal partitioning method:
  </td></tr>
</table>

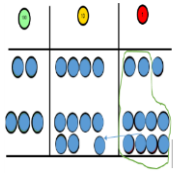
			Place larger number in your head and add on 10 more by counting in tens. Represent the number sentence in different ways: 41 = 31 +10, 31 + 10 = 41
TO + O **(No regrouping)**	Continue to develop understanding of place value and partitioning e.g. 41 + 8 Using dienes or Cuisenaire rods to show bar models.	Represent base 10 with lines / dots e.g. 41 + 8  The part whole model:  The bar model:  	Children to use **informal partitioning method:**  Introduction of the **partitioning column method:** 
TO + O **(With regrouping in the ones)**	Continue to develop understanding of partitioning and place value 36 + 5 Using dienes or Cuisenaire rods to show bar models.	Represent base 10 with lines / dots e.g. 36 + 5 The part whole model:  	Children to use **informal partitioning method:** 

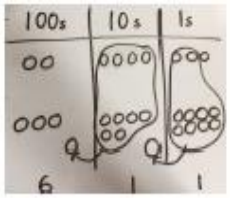
		36 5 The bar model: <table><tr><td colspan="2">?</td></tr><tr><td>36</td><td>5</td></tr></table>	Introduction of the **partitioning column method:** 
TO + TO **(No regrouping)**	Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters. E.g. 24 + 15   **Partitioning (Aggregation model)** 34 + 23 = 57 Base 10 equipment:  Children create the two sets with Base 10 equipment and then combine; ones with ones, tens with tens. Using dienes/Cuisenaires to show bar models.	After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions.  The part whole model:  The bar model: <table><tr><td colspan="2">?</td></tr><tr><td>24</td><td>15</td></tr></table>	Children to use informal **partitioning method:**  Introduction of **partitioning column method:** 

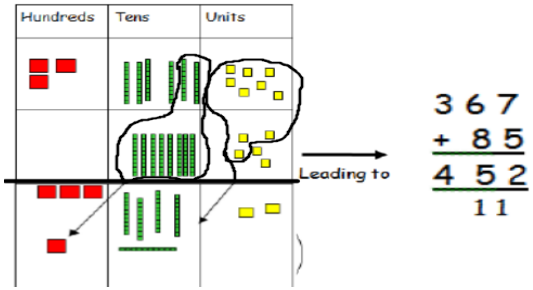
TO + TO **(With regrouping in the ones)**	Show regrouping using dienes: 36 + 25 37 + 15 = 52  What are the missing counters and digits problems. Use Cuisenaire rods to show bar models.	Children to represent base 10 by drawing it in a place  value chart. The part whole model:  The bar model: ? (36 | 25)	Children to use informal **partitioning method:**  Introduction of **partitioning column method:** 
Year 3			
HTO + O **(No regrouping)**	This can also be done with place value counters or Base 10. 134 + 215 =   Use Cuisenaire rods to show bar models.	Pictorial representation of the columns and counters.  The part whole model:  Bar models to show children what to do in a sum. ? (sum) (360 | 25)	Recap the **partitioning column method.** Introduce the **expanded column method.**  Regrouping: 
HTO + TO **(No regrouping)**			
HTO + TO **(With regrouping in the tens)**			
HTO + TO **(With regrouping in ones & tens)**			

Year 4/5/6

Y4 – **compact column method** up to 4 digits and two decimals (introduced with money) with the same number of digits. If there are more than 10 or more counters in a column regroup into a new counter in the next column. Then add up all the columns. Make it, Draw it, Write it.







3 6 7
+ 8 5
4 5 2
1 1

Y5 – **compact column method** with more than 4 digits and decimals with different number of digits and regrouping in some columns.
Make it, Draw it, Write it.

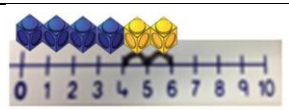
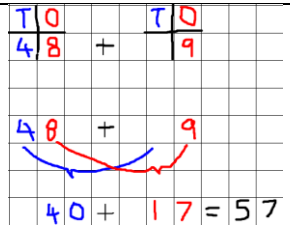
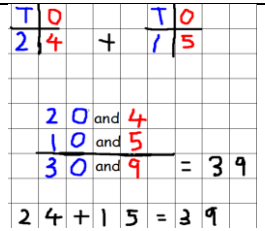
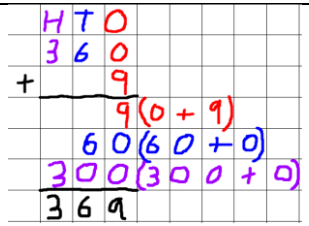
	£	2	3	.	5	9
+	£		7	.	5	5
	£	3	1	.	1	4
		1	1		1	

Use place value counters for adding decimals

Y6 – **compact column method** with more than four digits and decimals with different place value and regrouping in some columns.
Make it, Draw it, Write it.

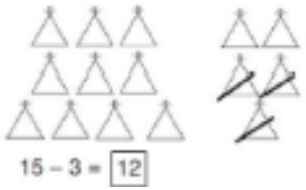
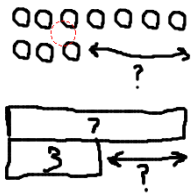
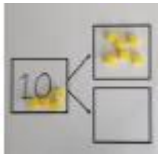
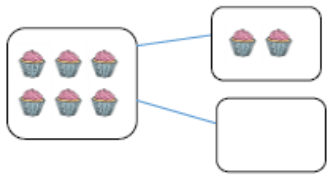
2	3	.	3	6	1
	9	.	0	8	0
5	9	.	7	7	0
+	1	.	3	0	0
9	3	.	5	1	1
2	1		2		

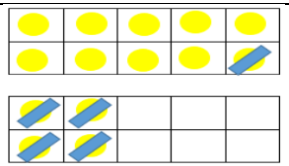
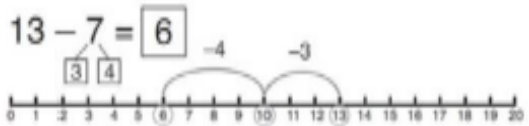
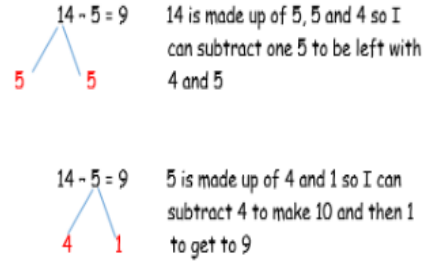
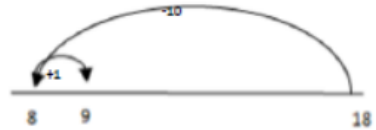
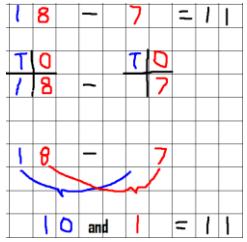
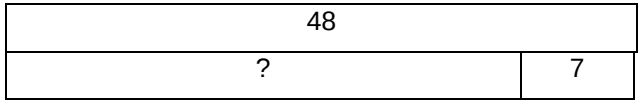
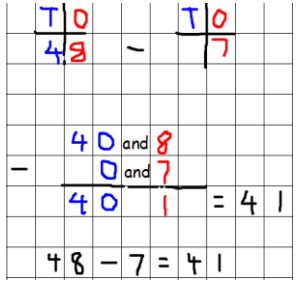
Use place value counters for adding decimals

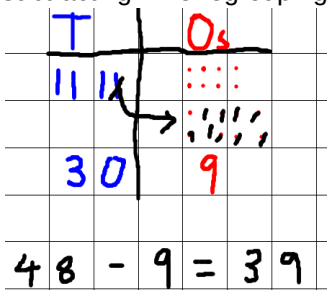
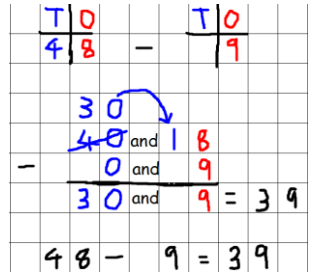
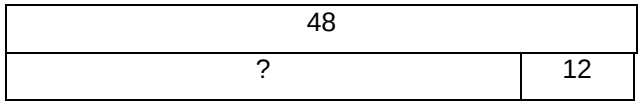
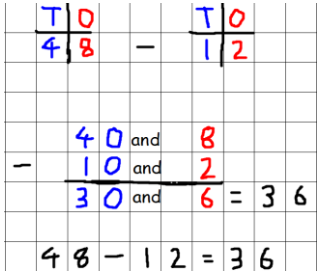
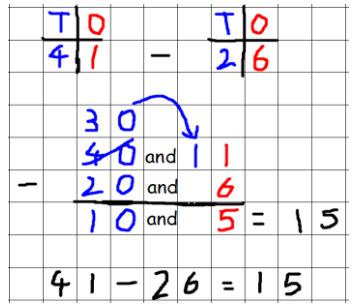
Quick Glance Addition Written Methods:		
Year Group	**Written Method Name**	**Written Method Example**
EYFS	Number tracks and Number lines	
Year 1	Informal Partitioning	
Year 2	Partitioning column	
Year 3	Expanded column	
Year 4/5/6	Compact column	243 +368 611 1 1

Quick Glance Addition Number Size	
Year Group	**Number size**
EYFS	Up to 1 digit + 1 digit
Year 1	Up to 2 digits + 1 digit
Year 2	Up to 2 digits + 2 digits
Year 3	Up to 3 digits (1000)
Year 4	Up to 4 digits including two decimal places
Year 5	More than 4 digits and decimals
Year 6	More than 4 digits and decimals

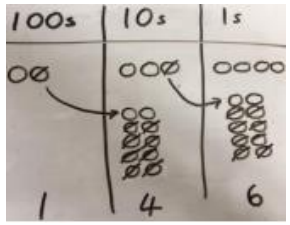
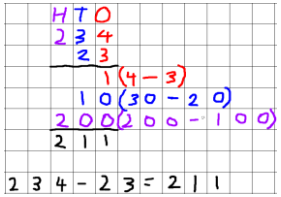
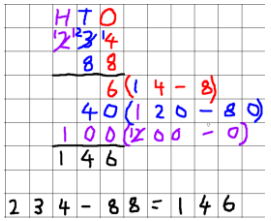
Subtraction

Objective and strategies	Concrete	Pictorial	Abstract
EYFS/Year 1			
Taking away ones	Use physical objects: counters, cubes etc to show how objects can be taken away (separation model).	Cross out drawn objects to show what has been taken away  	Children start to show recognisable abstract number sentences. 4 – 3 = 1
Counting back	Using number lines or number tracks. Children start with 6 and count back 2	Represent on number line (full and empty).	Children start to show recognisable abstract number sentences. 6 – 2 = 4
Find the difference	Finding the difference using cubes, bead strings Numicon or Cuisenaire rods (comparison model).	Draw the cubes. Use the bar model.  	Find the difference between 8 and 5 8 – 5 = Explore why 9 – 6 = 8 - 5
Part whole model	Link to addition – use the part whole model to help explain the inverse. Explore using counters and bead string.  	Use a pictorial representation of objects 	Use numbers within the part whole model 

Make 10 (bridging 10) by partitioning one of the numbers	14-5 (Numicon, counters, 10 square, bead string)  Take away 4 to make 10 Then takeaway 1 so you have taken away 5. You are left with the answer of 9.	Ten frame:  Number line. Start at 13. Partition the next number. Take away 3 to reach 10. Take away 4. 	Children start to show recognisable abstract number sentences. 13 - 7 = 6, 13 – 6 = 7 
Subtracting 10 and then compensating	18 – 9 Bead string:  Children find 18, then subtract 10 and then adjust by adding 1.	Children draw a picture to show the compensation. Number line: 	18 – 10 = 8, 8 + 1 = 9 Introduce **informal partitioning method:** 
Year 2			
TO – O (No regrouping)	Create the bigger number using base 10/place value counters and then subtract the smaller number. 48 – 7 Children should be advised to use mental methods to calculate this sum initially, before proving their answer with written methods.	Draw the base 10/place value counters and then cross out what you are subtracting. The bar model: 	Introduction of the **partitioning column method:** 

TO – O (With regrouping in the ones)	Create the bigger number using base 10/place value counters and then subtract the smaller number. You can't remove 9 from 8, so you need to 'steal' a ten from the next column. Regroup into 30 and 18. Children can play around with numbers that can add to 48. 48 – 9	Draw the base 10/place value counters and then cross out what you are subtracting. The regrouping must be clearly shown. 	Introduction of the **partitioning column method:** 
TO – TO (No regrouping)	Create the bigger number using base 10/place value counters and then subtract the smaller number. 48 – 12	Draw the base 10/place value counters and then cross out what you are subtracting. The bar model: 	Introduction of the **partitioning column method:** 
TO – TO (With regrouping in the ones)	Create the bigger number using base 10/place value counters and then subtract the smaller number. 41 - 26	Draw the base 10/place value counters and then cross out what you are subtracting. The regrouping must be clearly shown. 41 - 26	Introduction of the **partitioning column method:** 

Year 3

HTO - O (No regrouping)		Represent the place value counters pictorially; remembering to show what has been regrouped. 234-88  The bar model: 234 ? | 88	Introduce the **expanded column method.**  With regrouping: 234 – 88 
HTO - O (With regrouping)			
HTO - TO (No regrouping)			
HTO - TO (With regrouping in the tens)			
HTO - TO (With regrouping in ones & tens)			

Year 4/5/6

Y4 – compact column method with regrouping with up to 4 digits. Make it, Draw it, Write it.

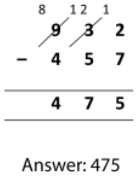
	12	123	4
-		8	8
	1	4	6

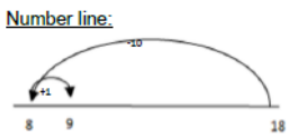
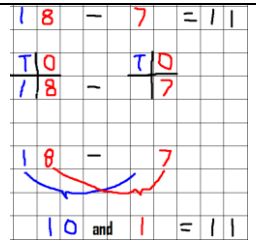
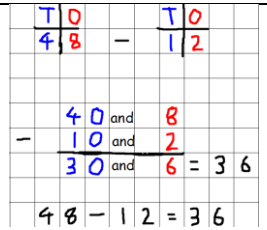
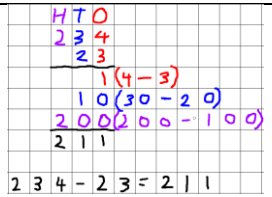
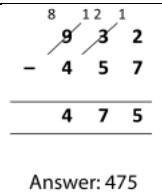
1

Y5 – compact column method with regrouping with more than 4 digits. Make it, Draw it, Write it.

	2	56	123	.	10
-		2	6	.	5
	2	3	6	.	5

Y6 - compact column method with regrouping with more than 4 digits. Place value counters for decimals with different amount of decimal places. Make it, Draw it, Write it.



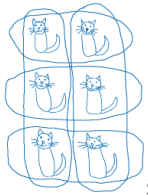
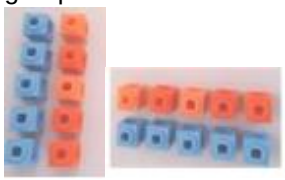
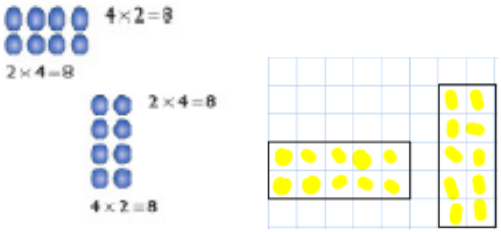
Quick Glance Subtraction Written Methods:		
Year Group	**Written Method Name**	**Written Method Example**
EYFS/Year 1	Number tracks and Number lines	
Year 1	Number lines and Informal Partitioning	
Year 2	Partitioning column	
Year 3	Expanded column	
Year 4/5/6	Compact column	

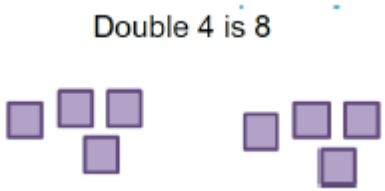
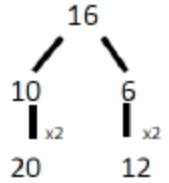
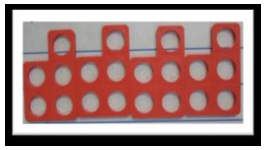
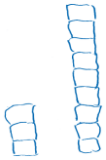
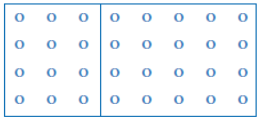
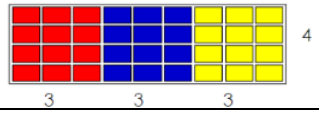
Quick Glance Subtraction Number Size	
Year Group	**Number size**
EYFS	Up to 1 digit - 1 digit
Year 1	Up to 2 digits - 1 digit
Year 2	Up to 2 digits - 2 digits
Year 3	Up to 3 digits (1000)
Year 4	Up to 4 digits including two decimal places
Year 5	More than 4 digits and decimals
Year 6	More than 4 digits and decimals

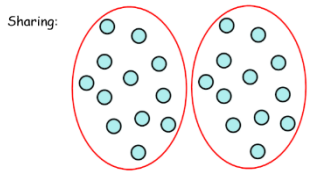
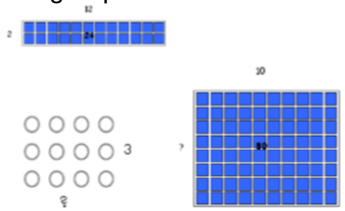
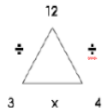
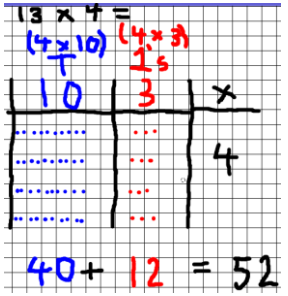
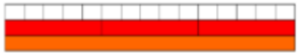
Multiplication

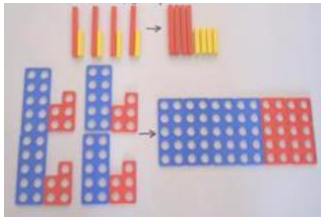
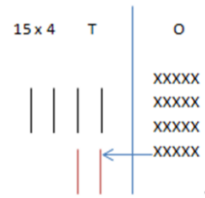
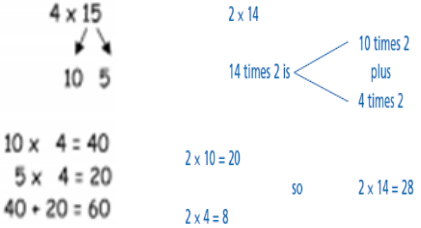
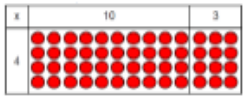
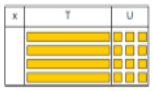
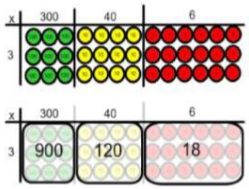
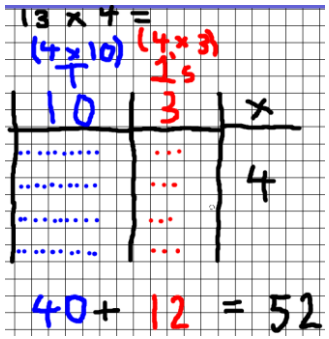
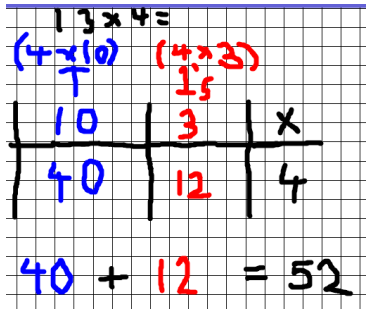
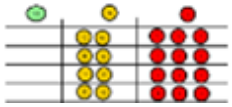
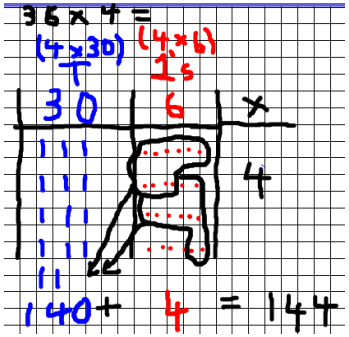
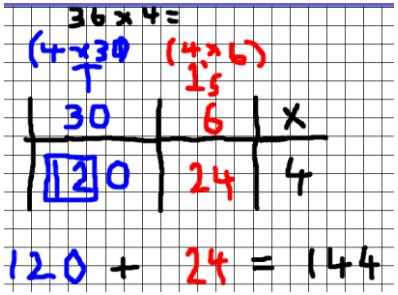
Multiplication and division are connected. Both express the relationship between a number of equal parts and the whole.

Part	Part	Part	Part
Whole			

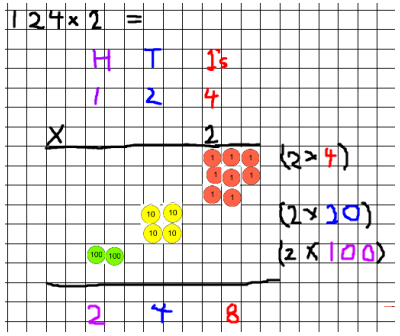
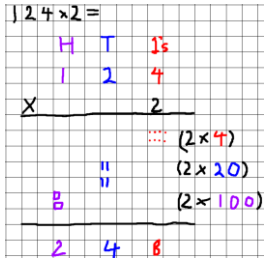
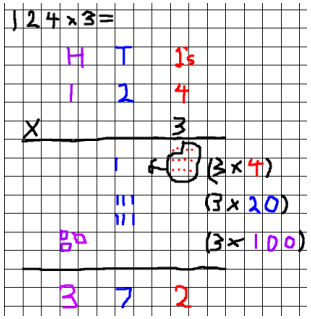
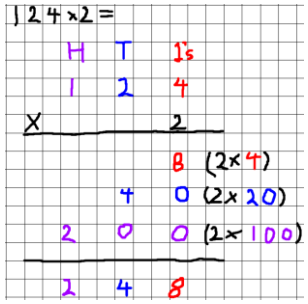
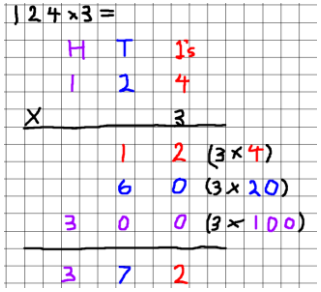
Objectives and strategies	Concrete	Pictorial	Abstract
EYFS/Year 1			
Multiply by adding equal groups together	Use a set of objects. Double the set by finding the same number again. Make sure both sets are equal.	 Draw the objects  showing: 2 x 3 and 3 x 2	Children may start to show recognisable abstract number sentences. 3 x 4 = 12 4 + 4 + 4 = 12
Introduction of using arrays to count in multiples of 2, 5, 10 (communtative law)	Use a set of objects. Children can place them in groups or start to focus them in on array shapes.  2 x 6 6 x 2	Draw the objects in arrays. Draw in different rotations to find the communtative sentences. This prepares children for the grid method and finding of factors. Also, to help find the area of rectangles. 	Children count in multiples of a number out loud. (See mental mathematics policy for more information). Write sequences with multiples of numbers. 2, 4, 6, 8 etc Children start to use an array to write a range of abstract calculations. 2 x 5 = 10, 5 x 2 = 10, 5 + 5 = 10, 2 + 2 + 2 + 2 + 2 = 10
Reason about odd and even numbers and relate to	Create arrays of odd and even numbers with objects – what is the same or different about them?	Draw the objects and circle/highlight the differences and similarities. Draw what happens when you double the number.	Children may start to show abstract number sentences.

doubling and halving	Double the number by adding the same number of objects and discuss what happens.		3 + 3 = 6 Odd + Odd = Even
Doubling of all numbers up to 10/ **halving**	Use practical activities to show how to double a  number.	Draw pictures to show how to double a number  	 Partition a number, then double each part before recombining
Repeated grouping / repeated addition	There are 3 equal groups with 4 in each group. Use a bead string to show repeated addition.  Children use Cuisenaire Rods to partition totals into equal trains.  Using Numicon to show 4 x 5: 	Make a necklace with red and yellow beads using three red beads for every yellow bead. Use the bricks to make a tower three times as high as this one:  Children represent the practical resources in a picture and use a bar model. Represent on the number line.	Children start to show recognisable abstract number sentences.  Children are taught about the multiplication 'x' symbol. 3 x 4 = 12 is the same as 4 + 4 + 4 = 12
Year 2			
Consolidating use of arrays and repeated addition (distributive law)	32 pegs on a board are to be arranged into fours. How can these be shown? This shows the distributive law where 8 x 4 = 3 x 4 + 5 x 4. 	Ch to illustrate this in different ways and should be encouraged to be flexible with how they use number and can be encouraged to break the array into more manageable chunks.  	Which could also be seen as 9 x 4 = (3 x 4) + (3 x 4) + (3 x 4) = 12 + 12 + 12 = 36 Or 3 x (3 x 4) = 36

Linking multiplication and division through missing number questions	Use objects to make 24. I know there are 2 lots so split them up. How many in each group? 2 x ? = 24 	Drawing arrays or groups: 3 X ? = 12 	Introducing the Inverse operations Trios can be used to model the 4 related multiplication and division facts. 3 x 4 = 12 4 x 3 = 12 12 ÷ 3 = 4 12 ÷ 4 = 3  Children use symbols to represent unknown numbers and complete equations using inverse operations. They use this strategy to calculate the missing numbers in calculations.
TO x O (No regrouping)	Use different resources to create the arrays.	Starting to organise and therefore draw arrays in columns  and show in a bar model. 52 4 4 4 4 4 4 4 4 4 4 4 4 4	
Year 3			
Multiplying factor (scaling)	Increasing a number of objects by a scale factor not by a fixed amount. For example, where you have 3 giant marbles and you swap each one for 5 of your friend's small marbles, you will end up with 15 marbles. This can be written as: 1 + 1 + 1 = 3 □ scaled up by 5 □5 + 5 + 5 = 15 	Children draw the word problem to find the solution. For example, find a ribbon that is 4 times as long as the blue ribbon.  We should also be aware that if we multiply by a number less than 1, this would correspond to a scaling that reduces the size of the quantity. For example, scaling 3 by a factor of 0.5 would reduce it to 1.5, corresponding to 3 x 0.5 = 1.5.	Children show recognisable number sentences. 5 x 4 = 20

Partition to multiply	Use Numicon to show 15 x 4  Show multiplication of the 10s and the 1s separately	Children represent the concrete manipulative in a picture  or	Children show the steps they have taken: 
TO x O (No regrouping)	1) Show the link with arrays with unifix cubes 13 x 4  4 rows of 10 4 rows of 3  4 rows of 13 2) Using Dienes in a grid 3) Using place value counters  = 1038	Children can represent the work they have done in a way that they understand.  Show in a bar model. 52 4 4 4 4 4 4 4 4 4 4 4 4 4	Introduction of the **grid method:** 
TO x O (with regrouping of ones into tens)	4 x 23 using place value counters (regrouping). Step 1: Make 4 lots of 23 under place value headings.  Step 2: I have 12 counters in the 1's column. Regroup 10 of these into the Ten's column. Step 3: Count the number in each column.	Children can represent the work they have done in a way that they understand. 	Introduction of the **grid method:** Discuss how multiplying 4 x 3 gives you 12 and the answer is ten times bigger. 

Year 4

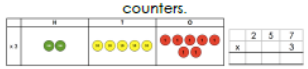
TO x O (no regrouping) **TO x O (extra digit in the answer)** **TO x O (with regrouping of ones into tens)**	Using place value counters to create the sum.  	Children to represent the counters / base 10 pictorially:  With regrouping of 1s into Tens, using Base 10:  Show in a bar model. 372 124 | 124 | 124	Introduction of **expanded short multiplication:**  With regrouping of 1s into Tens: 
HTO x O (with no regrouping) **HTO x O (with regrouping of ones to tens)** **HTO x O**		Children to represent the counters / base 10 pictorially:	Introduction of **compact short multiplication:**

(with regrouping of tens into hundreds)

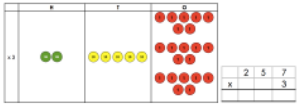
HTO x O (with regrouping of ones into tens and tens into hundreds)

257 x 3 =
Use the place value counters to demonstrate multiplying in columns.

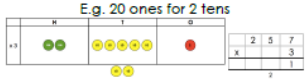
Make the number with the place value counters.



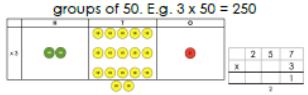
Start with the ones. Make 3 groups of 7. E.g. 3 x 7 = 21



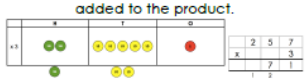
If there are 10 or more counters in a column exchange for counters in the next highest column.
E.g. 20 ones for 2 tens



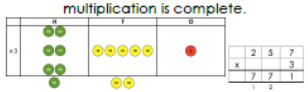
Now look at the tens, children make 3 groups of 50. E.g. 3 x 50 = 250



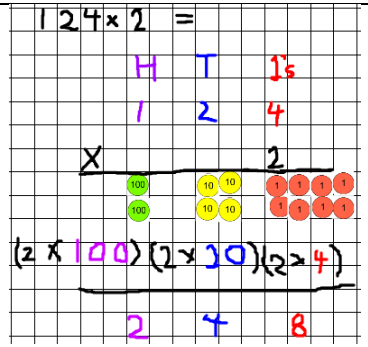
If there are 10 or more counters in a column exchange for the next highest column.
E.g. 10 tens for 1 hundred
The previously exchanged counters are added to the product.



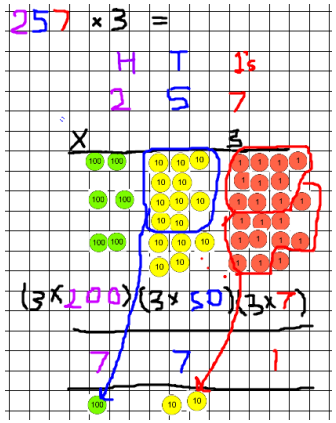
Now look at the hundreds, make 3 groups of 200. E.g. 200 x 3 = 600
The previously exchanged counters are added to the product and the multiplication is complete.



It is important at this stage that they always multiply the ones first and note down their answer followed by the tens, then hundreds etc.

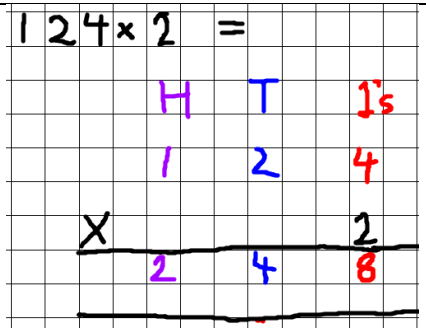


With regrouping of 1s into Tens (place value counters):

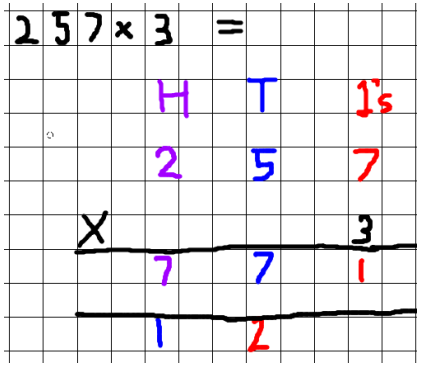


Show in a bar model.

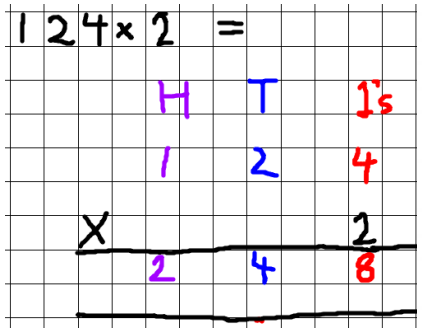
771		
257	257	257

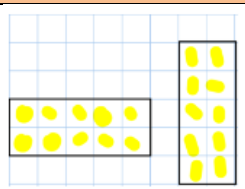
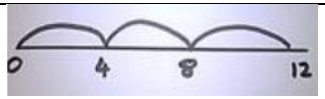
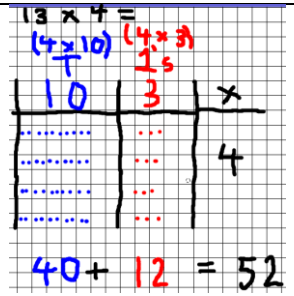
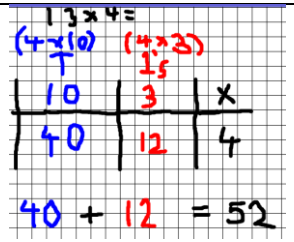


With regrouping:

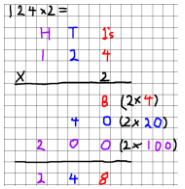
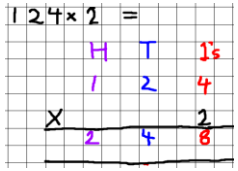
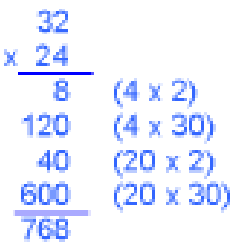
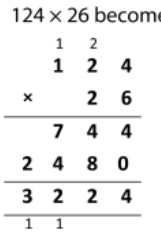


Year 4/5/6

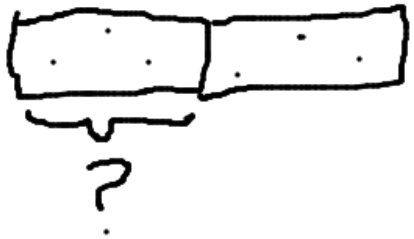
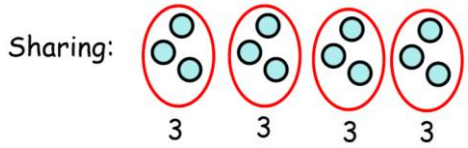
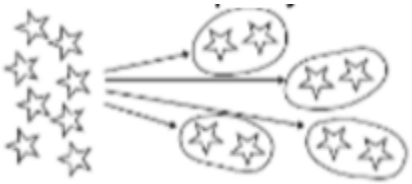
Y5 - Introduction of **expanded long multiplication** with up to 4 x 2 digits. Make it, Draw it, Write it. 32 x 24 8 (4 x 2) 120 (4 x 30) 40 (20 x 2) 600 (20 x 30) 768	**Y5** - Introduction of **compact long multiplication** with up to 4 x 2 digits**.** Make it, Draw it, Write it. 124 × 26 becomes 1 2 1 2 4 × 2 6 7 4 4 2 4 8 0 3 2 2 4 1 1	**Y6** – Consolidation of **compact short multiplication** and **compact long multiplication** methods with up to 4 digits by a 2 digit. Including multiplication of decimals. Make it, Draw it, Write it.  124 × 26 becomes 1 2 1 2 4 × 2 6 7 4 4 2 4 8 0 3 2 2 4 1 1

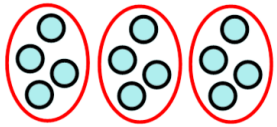
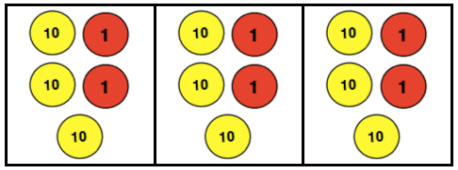
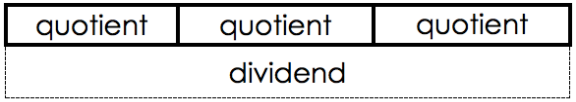
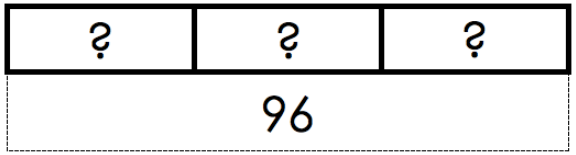
Quick Glance Multiplication Written Methods:		
Year Group	**Written Method Name**	**Written Method Example**
EYFS	Arrays	
Year 1	Arrays and repeated addition	
Year 2	Arrays in a grid	
Year 3	Grid method	

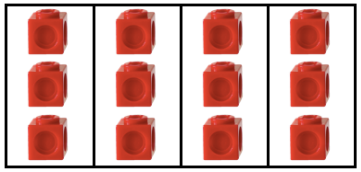
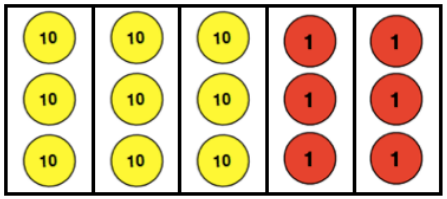
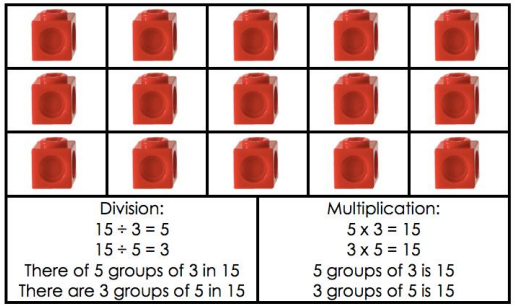
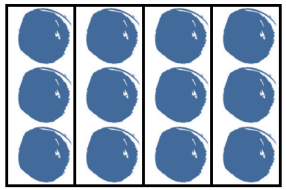
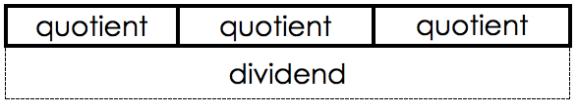
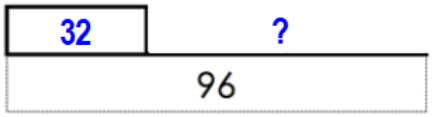
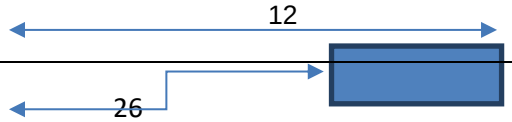
Quick Glance Multiplication Number Size (Children must stay within these boundaries)	
Year Group	**Number size**
EYFS/Year 1	Up to 2 digits x 1 digit
Year 1	Up to 2 digits x 1 digit
Year 2	Up to 2 digits x 1 digit
Year 3	Up to 2 digits x 1 digit
Year 4	Up to 3 digits x 1 digit
Year 5	Up to 4 digits x Up to 2 digits
Year 6	Up to 4 digits x Up to 2 digits

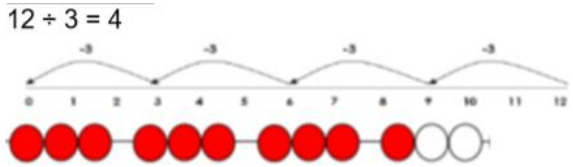
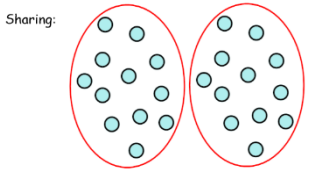
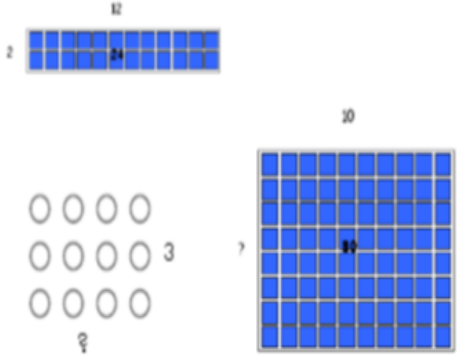
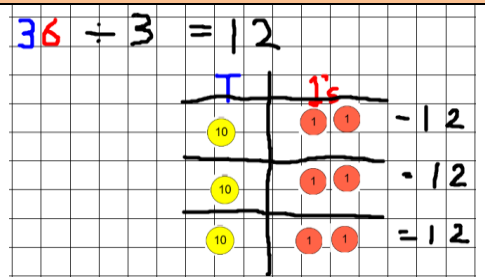
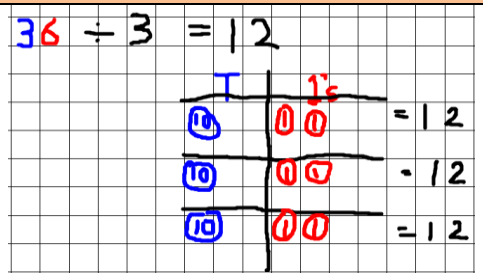
Quick Glance Multiplication Written Methods:		
Year Group	**Written Method Name**	**Written Method Example**
Year 4	Expanded short multiplication and compact short multiplication	**Expanded Short:** **Compact Short:**  
Year 5 **Year 6**	Expanded long multiplication and compact long multiplication	**Expanded Long:** **Compact Long:**  

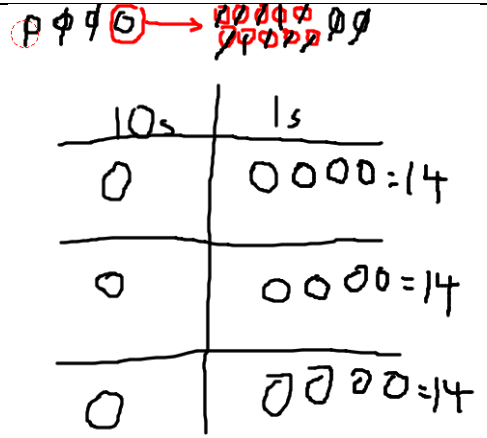
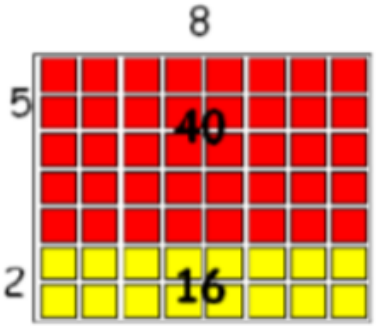
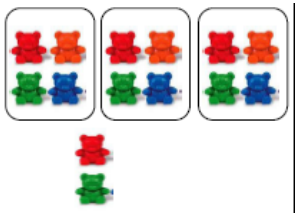
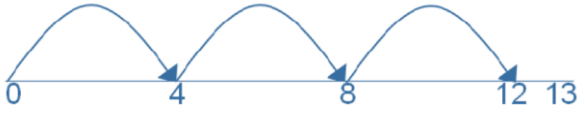
Division

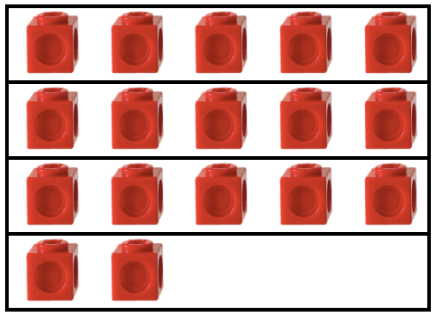
Objectives and strategies	Concrete	Pictorial	Abstract
	EYFS/Year 1		
Equal groups	Children will group different objects into equal sized piles.	Children will draw equal sized groups of objects.	
Sharing objects into equal sized groups	I have 6 cubes; can you share them equally into 2 groups? 	Represent the idea pictorially and using a bar. 	6 ÷ 2 = 3 Children should be encouraged to link these ideas to their times tables facts. Ch could draw bars with abstract numbers in them.
Solve problems which involved sharing or grouping **Know all halves to 10 through grouping and sharing**	Sharing: Introduce practical problems which the children can physically solve. 1) Look at the number that we are dividing e.g. 12 2) Share this number out equally into section of the number that we are dividing by e.g. 4 3) Count how many there are in each section 6 sweets get shared between 2 people. How many sweets do they each get? A bottle of fizzy drink shared equally between 4 glasses. How much does each person get? 12 ÷ 4 = 3  Grouping:	Draw a picture to show what happened. Sharing:  Grouping: 	

	1) Look at the number that we are dividing e.g. 12 2) Count or draw this many objects 3) How many groups of the number we are dividing by (e.g. take 4 objects and make one group) can you make? 4) Count how many groups you have made There are 6 sweets. How many people can have 2 sweets each? 12 ÷ 4 = 3 Grouping: 		
Year 2			
Sharing objects into groups	Share objects into groups. I have 12 cubes.  Can they be shared equally in 3 groups? After sharing between 3 groups we have found that are 4 in each group.  12 ÷ 3 = 4 Using place value counters e.g. 96 ÷ 3 = 32  After sharing we found there were 3 tens and 2 ones in each group.	Use pictures or shapes to share quantities.  12 ÷ 3 = 4 Bar Modelling: Split the bar into the number of groups you are dividing by and work out how many would be within each group. Children do not need to use these words! no. of boxes = divisor  dividend ÷ divisor = quotient e.g. 96 ÷ 3 = 	Share 12 sweets between 3 people. 12 ÷ 3 = 4 Share £96 between 3 children. £96 ÷ 3 = £32

Grouping objects	Divide quantities into equal sized groups. I have 12 cubes.  After making groups of 3 we discovered there were 4 of them.  12 ÷ 3 = 4 Using place value counters e.g. 96 ÷ 3 = 32  After making groups of 3, we find there were 3 groups of ten and 2 groups of one. Creating different arrays using cubes.  Division: 15 ÷ 3 = 5 15 ÷ 5 = 3 There of 5 groups of 3 in 15 There are 3 groups of 5 in 15 Multiplication: 5 x 3 = 15 3 x 5 = 15 5 groups of 3 is 15 3 groups of 5 is 15	Represent using arrays: How many strawberries will each child have if 30 are shared between 5 children? 30 ÷ 6 = 5  30 ÷ 5 = 6 Arrays are really important as they link to the bus stop method!  12 ÷ 3 = 4 Bar Modelling: You know how many would be within each group, but need to find out how many groups. no. of boxes = divisor  dividend ÷ quotient = divisor 96 ÷ ? = 32  	Sweets are sold in bags of 3. If I have 12 sweets how many bags would I need? 12 ÷ 3 = 4 There are 96 children sitting in rows of 3. How many rows are there? 96 ÷ 3 = 32
Grouping using repeated subtraction	Using Cuisenaire rods above a ruler. Discuss that the number sentence (6 ÷ 2 = ?), says, "How many 2s fit into 6?" How big is each hop/rod?	Represent using a bar model and link to the Cuisenaire rods and bead strings: 12 ÷ ? = 3  	Children are introduced to the ÷ sign. 12 ÷ 4 = 3 12 ÷ 3 = 4 This is linked to the Number line.

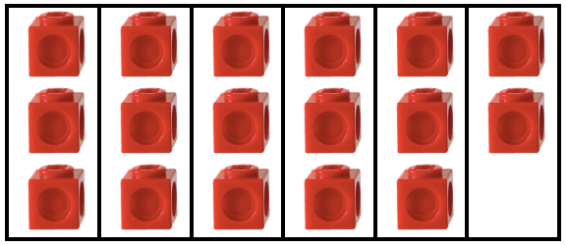
	Use a bead string to help children to group. 	3 ? Represent in a Number line to show the equal groups that have been subtracted. The arrows go from the dividend to zero. The number of jumps equals the number of groups.	12 – 4 – 4 – 4 = 0 12 – 3 – 3 – 3 – 3 = 0 Discuss how division is not commutative e.g. 12 ÷ 3 = 4 but 3 ÷ 12 doesn't = 4 However, 12 ÷ 3 = 4 and 12 ÷ 4 = 3!
Linking multiplication and division through missing number questions	Use objects to make 24. I know there are 2 lots so split them up. How many in each group? 2 x ? = 24 	Drawing arrays or groups: 3 X ? = 12 	Introducing the Inverse operations Trios can be used to model the 4 related multiplication and division facts.  3 x 4 = 12 4 x 3 = 12 12 ÷ 3 = 4 12 ÷ 4 = 3 Children use symbols to represent unknown numbers and complete equations using inverse operations. They use this strategy to calculate the missing numbers in calculations.
Year 3			
Sharing **TO ÷ O (with no regrouping and no remainder)**			36 ÷ 3 = 12

Sharing **TO ÷ O (with regrouping and no remainder)**	42 ÷ 3 = 14		Children to be able to make sense of the place value counters and write calculations to show the process. 42 ÷ 3 42 = 30 + 12 30 ÷ 3 = 10 12 ÷ 3 = 4 10 + 4 = 14
Using the distributive law	Create arrays using cubes. Model grouping the sums into different colours or sections. E.g. 56 ÷ 8 can be done as 40 ÷ 8 and 16 ÷ 8	Ch draw the pictorial representation of the array of  cubes.	Write their sum using abstract numbers. 56 ÷ 8 = 7
Sharing **TO ÷ O (no regrouping and a remainder)**	Use concrete objects to share between groups to see how many is left over. 14 ÷ 3 = 4 r 2  Sharing: 17 ÷ 3 = 5 r 2 Divide objects between groups and see how much is left over.	Use a number line to see how many more you need to jump to find a remainder. 13 ÷ 4 = 3 r 1  Draw dots in circles to divide an amount and clearly show a remainder. Sharing: 17 ÷ 3 = 5 r 2  Grouping: 17 ÷ 3 = 5 r 2	Complete written divisions and show the remainder using r. 



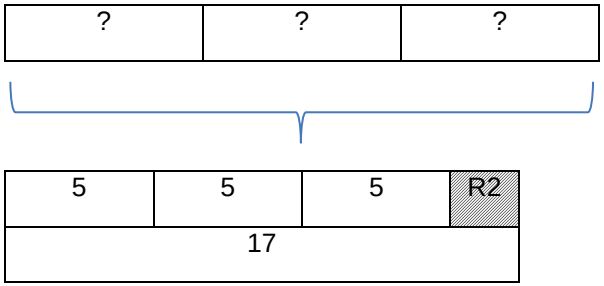
There are 5 in each group with 2 remaining.

Grouping 17 ÷ 3 = 5 r 2
Put objects into groups of 3 and see how many are left over.



There are 5 groups and with 2 remaining.

Bar model can be used to display whole being divided into equal parts with a remainder left over.
17 ÷ 3 = 5 r 2



Year 4

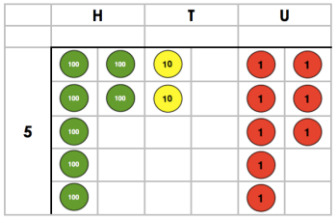
TO ÷ O (No regrouping and no remainder)

TO ÷ O (No regrouping and a remainder)

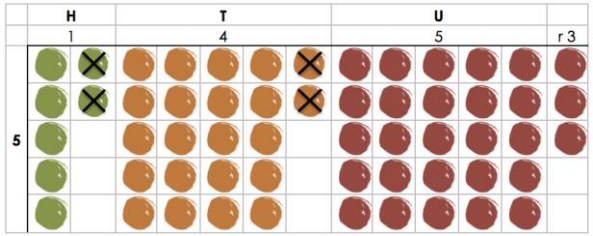
TO ÷ O (Regrouping and no remainder)

TO ÷ O (Regrouping and a remainder)

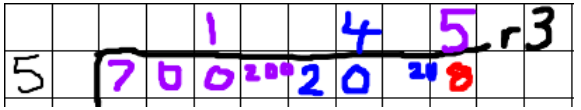
Alongside using counters children to see the expanded short division method.



Draw the counters and cross out any that are regrouped.

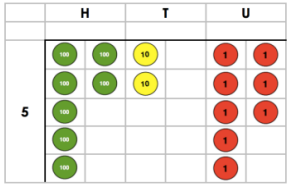
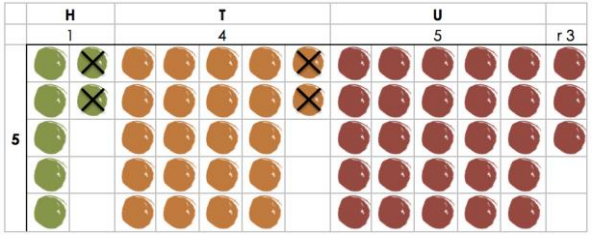
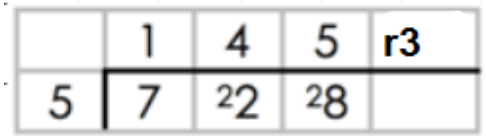


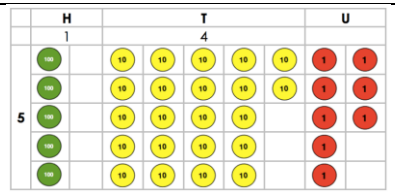
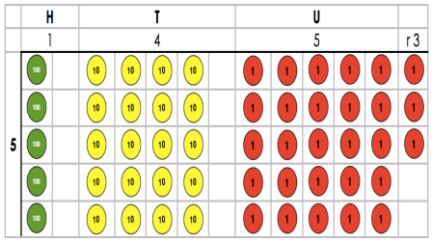
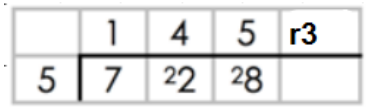
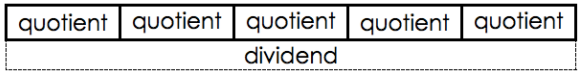
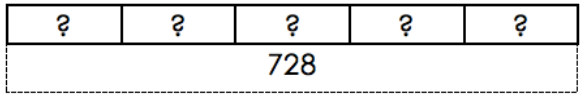
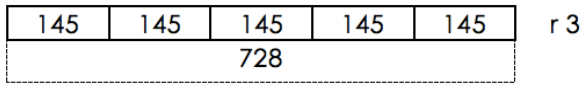
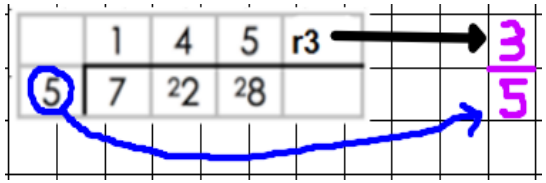
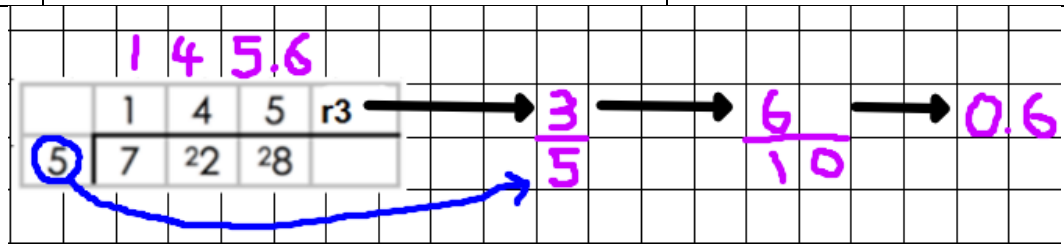
Could introduce the **expanded short division method:**



Discuss how you are trying to find out how many groups of 5 the 700 can be placed in, not how many 5s fit into the number 700.

Model alongside the concrete manipulatives and the pictorial representation.

			Or, use compact short division.
HTO ÷ O (No regrouping and no remainder)			
HTO ÷ O (No regrouping and a remainder)			
HTO ÷ O (Regrouping of hundreds into tens)			
HTO ÷ O (Regrouping of tens into ones)			
HTO ÷ O (Regrouping of hundreds into tens and tens into ones and a remainder)			
Year 5			
Where there are zeros in the quotient e.g. 816 ÷ 4=204	728 ÷ 5 =145r3 Layout the bus stop. Place counters in an array with the number of rows reflecting the divisor & keeping to column values.  Start with the biggest place value. 7 hundreds put into groups of 5. 1 group of 5 with 2 hundreds left over. Regroup these 2 hundreds for 20 tens. These are then placed within the array in the tens column.	Draw a pictoral representation of the columns and place value counters.  Children should only draw in one column at a time. Symbols can be crossed out when regrouping takes place. Bar modelling can support learners when	Introduction of the **compact short division method:**  Model alongside the concrete manipulatives.
HTO ÷ O no regrouping and no remainder			
HTO ÷ O no regrouping and a remainder			
HTO ÷ O with regrouping of hundreds into tens and no remainder			

HTO ÷ O with regrouping of tens into ones and no remainder **HTO ÷ O with regrouping and a remainder** **HTO ÷ O with regrouping of tens into ones and a remainder** **HTO ÷ O with regrouping hundreds into tens and a remainder** **HTO ÷ O with regrouping of hundreds into tens and tens into ones and a remainder**	 22 tens put into groups of 5 would be 4 tens with 2 tens remaining. These remaining counters are regrouped once again.  Finally, 28 ones, grouped into 5 would 5 with a remainder of 3. 	solving problems with division alongside the formal written methods. no. of boxes = divisor  dividend ÷ divisor = quotient  728 ÷ 5 = 145 r 3  r 3
Remainder expressed as a fraction **Remainder expressed as a simplified fraction** **Remainder expressed as a decimal**	 	

Year 6

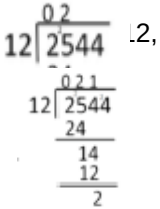
Long division

(2 digit divisors)

Long division using place value counters
2544 ÷ 12

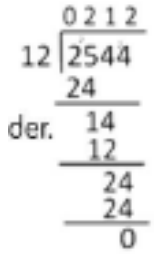
We can't group 2 thousands into groups of 12 so we will exchange them into the hundreds column.

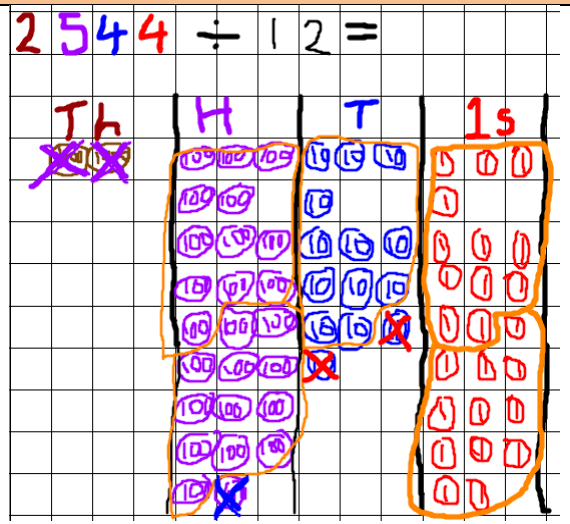
We can group 24 hundreds int .2, which leaves 1 hundred



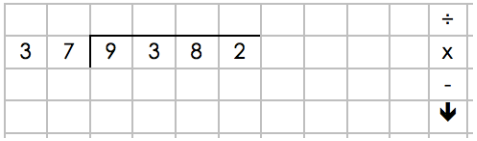
After exchanging the hundred, we have 14 tens. We can group 14 tens into one group of 12, which leaves 2 tens.

After exchanging the 2 tens, we have 24 ones. We can group 24 ones into 2 groups of 12, leaving no remainder

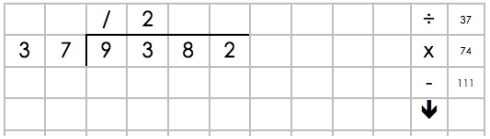




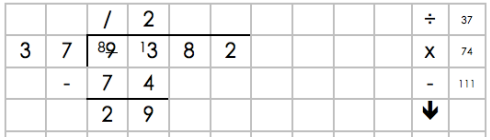
9382 ÷ 37
Encourage children to write the four steps (divide, multiply, subtract and bring down) as checklist.



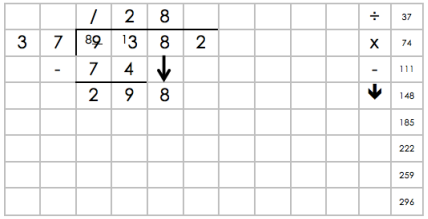
Divide the first 2 digits of the dividend by the divisor. Encourage children to jot the multiples of the divisor on the side to check.



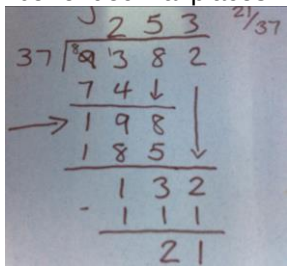
In short division the children would be working out the remainder to exchange mentally. Ensure the children understand they are doing the same thing but recording it on the paper, as the number is larger. This is done multiplying the divisor by quotient (E.g. 2 x 37) and then subtracting this from the dividend.



Rather than writing the remainder in the next column (Short division) the next column is brought down to the remainder.



This process is repeated until each column has been divided leaving a remainder or to

			a certain number of decimal places. 

Quick Glance Division Written Methods:		
Year Group	**Written Method Name**	**Written Method Example**
EYFS **Year 1**	Sharing and grouping in circles	Sharing: 3 3 3 3 Grouping:
Year 2	Sharing and grouping in arrays	
Year 3	Sharing within place value columns	36 ÷ 3 = 12

Year Group	**Written Method Name**	**Written Method Example**
Year 4	Expanded short division, or compact short division	**Expanded short division:**
Year 5	Compact short division	**Compact short division:** 1 4 5 r3 5 | 7 22 28
Year 6	Long division	**Long division:**

Quick Glance Division Number Size (Children must stay within these boundaries)	
Year Group	**Number size**
EYFS/Year 1	Up to 2 digits ÷ 1 digit
Year 2	Up to 2 digits ÷ 1 digit
Year 3	Up to 2 digits ÷ 1 digit
Year 4	Up to 3 digits ÷ 1 digit
Year 5	Up to 3 digits ÷ 1 digit
Year 6	Up to 4 digits ÷ 2 digit

Mathematical Reasoning

Similar and different
- Comparing written methods
- Comparing numbers
- Comparing shapes
- Comparing properties of numbers
- Comparing representations

Odd one out
- Exploring mathematical language
- Exploring properties of numbers or shapes
- Exploring visualisations

True or false
- Exploring mathematical symbols
- Exploring vocabulary
- Exploring properties of number and shape
- Exploring equivalence
- Exploring greater than or less than

Spot the mistake
- Refining written methods
- Teaching procedural activities (e.g. reading a clock, a protractor)
- Exploring number relationships

What could this not be?
- Refining calculation
- Exploring misconceptions
- Estimation
- Procedural fluency
- Properties of shapes and numbers
- Algebra
- Mathematical vocabulary

Multiple ways
- Exploring relationships
- Building mathematical resilience
- Exploring number bonds
- Exploring quantity
- Exploring equivalence and equations

Hidden information
- Forging mathematical connections
- Building resilience
- Finding ways in to a problem
- Application of multiple skills

Conjectures
- Exploring misconceptions
- Developing vocabulary
- Instigating investigation
- Exploring exceptions to the rule
- Exploring patterns and relationships

Say what you see
- Exploring maths in different contexts
- Forging new relationships between numbers
- Developing vocabulary
- Promoting abstract visualisation

Working backwards
- Refining methods
- Forging relationships and connections
- Exploring number
- Exploring vocabulary
- Developing resilience

Reasoned estimations
- Develop number sense
- Understanding relative size
- Developing procedural method understanding
- Transition from pictorial representations

Mathematical arguing
- Using mathematical language

Which is the most difficult?
- Looking at bridging, exchanging
- Judging calculations on their complexity rather than number size.

Mental Maths Progression

	Autumn Term	Spring Term	Summer Term
Reception	• Count reliably to 20.	• Order numbers 1-20 • Say 1 more/1 less to 20	• Counting in 10's, 5's and 2's • Know doubles to 10 • Add and subtract two single digit numbers
Year 1	• Add and subtract within 5 • Subtract within 5 • Add within 5 • Add and subtract 1 to a 2 digit number • Subtract within 10 • Adding within 10 • Number bonds to 10	• Counting in 10's, 5's and 2's • Know halves of even numbers to 20 • Know doubles to 10 • Add and subtract 10 to a 2 digit number • Add 3 single digit number together • Use language of day, week, month and year. Tell time to hour and half past.	• Number bonds to 20 • Subtract any 1 digit number from any 2 digit number • Add any 1 digit number to any 2 digit number • Finding how many 'sets of' a smaller number make a bigger number Recognise half and quarter of an object, shape or quantity
Year 2	• Add any pair of 2 digit numbers • Add and subtract multiples of 10 to any give 2-digit number • Say 10 more/less than any number to 100 • Add two or three single digit numbers • Know all the pairs of numbers to 10, 12 and pairs with total of 20 • Count on and back in ones and tens from any given 2 – digit number	• Learn 2x, 5x, and 10x table (looking at lots of) • Double numbers up to 20 • Using fingers, say where a given number is in the 2s, 5s or 10s count (e.g. 8 is the fourth number when I count in twos • Count in 2s, 5s, and 10s • Subtract any pair of 2-digit numbers by counting back in tens and ones or by counting up	• Begin to double two-digit numbers less than 50 with digits of 1,2,3,4 or 5 • Double and begin to halve numbers to 40 and multiples of 10 and 100 Halve/Double numbers to 20 • Relate division to grouping (how many groups of five in fifteen) • Tell time to five minutes, including quarter past/to • Recognise half, 1/3, 2/4, ¾ of a shape, quantity or object • Begin to count in 3's and learn the 3x table.
Year 3	• Use place value and number facts to add and subtract numbers • Subtract by counting up • Learn to count in 3's and 4's and know the 3x and 4x table and relevant division facts • Add and subtract any two digit numbers by counting on in 10s and 1s or by using partitioning • Perform place value subtractions without a struggle (536-30=506) • Know multiples of 10 with a total of 100 • Know pairs with each total to 20	• Find 10 or 100 more/less than a given number. Count on in 50's from 0 • Tell the time to the nearest minute using 12 and 24 hour clocks, know the number of days in a month. • Subtract, when appropriate, by counting back or taking away, using place value and number facts • Learn to count in 9's and 8's and begin to learn 9x and 8x table and relevant division facts • Add and subtract pairs of 'friendly' 3 digit numbers, e.g. 230 +450	• Recognise fractions that add to 1. (e.g. ¼ +3/4) • Halve even numbers up to 100, halve add numbers to 20. • Double numbers up to 50 • Partition teen numbers to multiply by a single digit number (3 x 14 as 3 x10 and (3x4) • Begin to learn to count in 6's, 7's and 8's. Begin to know the 6x, 7x and 8x tables and relevant division facts

Year 4	• Find 1000 more/less than a given number. • Add and subtract £1, 10p and 1p to amounts of money. • Know by heart, quickly derive number bonds to 100 and £1 • Add and subtract any two 2 digit numbers by partitioning or counting on • Begin to learn to count in 11's and 12's. Begin to know the 11x, and 12x tables and relevant division facts	• Read and compare and convert between analogue/digital 12/24 hr clocks. • Multiply mentally one digit by two digit numbers • Count in 6's and 8's. Know 6x and 8x tables • Find change from £10, £20 and £50 • Count in multiples of 25 • All times tables facts	• Begin to double and halve amounts of money (£35.60 doubles = £71.20) • Read Roman numerals to 100. • Count up/down in hundredths • Partition 2-digit numbers to multiply by a single –digit number mentally (4 x 24 as 4 x 20 and 4 x 4) • Use understanding of place value and number facts in mental multi and division (36 x 5 is half of 36 x 10 and 50 x 60 = 3000 or 245 ÷20 is double 245 ÷ 10) • Divide multiples of 100 by 1-digit numbers using division facts (3200 ÷ 8= 400) • All times tables facts
Year 5	• Use place value and number facts to add two or more friendly numbers including money and decimals (e.g. 3+4+8+6+7, 0.6+0.4+0.7) • Add and subtract decimal numbers which are near multiples of 1 or 10 including money (e.g £6.34-1.99 or £34.59-£19.95) • Count in 11's and 12's and learn the 11x and 12x table • Add to the next 10 from a decimal number (e.g 13.6 + 6.4 =20). • Know number bonds to 1 and to the next whole number	• Use doubling and halving as mental division/multi strategies (58 x 5 = half of 58 x 10) • Use knowledge of factors and multiples in multiplication e.g (43 x 6 is double 43 x 3 and 28 x 50 is half of 28 x 100 = 1400) • Identify all multiples and factors including finding all factor pairs. • Know 3x,4x,6x,8x table. Apply and extend • Know square numbers and square roots up to 144. • Recall prime numbers up to 19 • All times tables facts	• Count up/down in thousands • Read Roman numerals to 1000. • Use knowledge of multiples and factors, test for divisibility (246 ÷ 6 = 123 ÷ 3) • Double and halve money by partitioning (Half of £75.40 = Half of £75 (37.50) plus half of 40p) • All times tables facts
Year 6	• Add two 1-place decimal numbers or two 2-place decimal numbers less than 1 (4.5 + 6.5 or 0.74 +0.33) • Count forward and backward with positive and negative numbers through zero. • Know all multiplication tables to 12x. Apply and extend • Derive quickly and without difficulty, number bonds to 1000 • Use number bonds to 1 and 10 to perform mental subtraction of any pair of one-place	• Use divisibility tests to aid mental calculation • Use place value and number facts in mental multi (40,000 x 6 = 24,000) • Identify common factors, common numbers and prime numbers and use factors in mental division (438 ÷ 6 is 219 ÷ 3) • Identify common factors, common numbers and prime numbers and use factors in mental multiplication (e.g 326 x 6 is 652 x 3) • Know by heart all multiplication and division facts up to 12 x 12. Apply and extend • Add positive number to negative numbers (e.g calculate a rise in temp)	• Halve and double decimal numbers with up to 2 places using partitioning e.g 36.73 doubled is double 36 plus double 0 .73) • Use rounding in mental multiplication (34 x 19 as (20 x34) -34) • Use doubling and halving as a mental division and multiplication strategy. E.g to divide by 2,4,8,5,20 and 25 (628 ÷ 8 is halved three times) (28 x 25 is ¼ of 28 x 100 = 700) • All times tables facts